Review for Test 2

1. Branching Processes

   $P_0, P_1, P_2, \ldots$ probabilities of i offspring; $\mu = \sum nP_n$
   X_n is the number of individuals at generation n; $X_0 = 1$
   want probability π_0 of extinction
   if $\mu \leq 1$ then $\pi_0 = 1$; otherwise ($\mu > 1$) then π_0 is the smallest positive root of the equation $x = P_0 + P_1 x + P_2 x^2 + \cdots$

2. Exponential Distribution

   $f(x) = \lambda e^{-\lambda x}$, $x > 0$; $\overline{F}(x) = \mathrm{P}(X > x) = e^{-\lambda x}$; $E[X] = 1/\lambda$
   memoryless property, constant failure rate λ
   sum of n exponential(λ) variables is a Gamma(n, λ)
   minimum of $X_1, X_2, \ldots, X_n$ is exponential($\lambda_1 + \lambda_2 + \cdots + \lambda_n$)
   $\mathrm{P}(X_1 < X_2) = \frac{\lambda_1}{\lambda_1 + \lambda_2}$

3. Poisson Process

   $N(t)$ is the number of events occurring in $(0, t)$; λ is the average number of events per unit time
   independent and stationary increments, $N(t)$ is Poisson(λt)
   interarrival times T_i are all exponential(λ)
   waiting time $S_n = T_1 + \cdots + T_n$ is Gamma(n,λ)
   modified Poisson (filtered by probability p)
   nonhomogeneous Poisson (with parameter m defined by $\int \lambda(t)\,dt$)
   $\mathrm{P}(S_n^A < S_m^B)$

4. CTMC

   state $X(t)$, Markov property
   parameters v_i, P_{ij}, transition rates $q_{ij} = v_i P_{ij}$
   examples: Poisson process, birth-death process, queues
   $P_{ij}(t)$ governed by the Chapman-Kolmogorov equations and (forward) Kolmogorov differential equations
   limiting probabilities
   - found by solving steady-state equations
   - sufficient conditions for existence
   - interpretations

   use limiting probabilities P_j to answer questions about system