Review for Final

1. Probability Concepts
   conditional probabilities
   Bayes’ formula
   independence

2. Random Variables
   standard distributions
   binomial, geometric, Poisson, exponential, gamma
   expected value, variance
   moment generating function
   found using conditioning

3. Markov Chains
   formulation of models, state transition diagram
   transition probabilities, higher-order transition probabilities
   absolute probabilities
   classification of states (recurrent, transient, periodic)
   limiting, stationary probabilities
   supporting theorems
   gambler’s ruin
   branching processes

4. Exponential Distribution
   \( f(x) = \lambda e^{-\lambda x}, \ x > 0; \ \bar{F}(x) = e^{-\lambda x}; E[X] = 1/\lambda \)
   memoryless property, constant failure rate \( \lambda \)
   sum of \( n \) exponential(\( \lambda \)) variables is a Gamma(\( n, \lambda \))
   minimum of \( X_1, X_2, \ldots, X_n \) is exponential(\( \lambda_1 + \lambda_2 + \cdots + \lambda_n \))
   \( P(X_1 < X_2) = \frac{\lambda_1}{\lambda_1 + \lambda_2} \)
5. Poisson Process

\( N(t) \) is the number of events occurring in \((0, t)\); \( \lambda \) is the average number of events per unit time

independent and stationary increments, \( N(t) \) is Poisson(\( \lambda t \))

interarrival times \( T_i \) are all exponential(\( \lambda \))

waiting time \( S_n = T_1 + \cdots + T_n \) is Gamma(\( n, \lambda \))

modified Poisson (filtered by probability \( p \))

nonhomogeneous Poisson (with parameter \( m \) defined by \( \int \lambda(t) \, dt \))

\( P(S_n^A < S_m^B) \)

6. CTMC

state \( X(t) \), Markov property

parameters \( v_i, P_{ij} \), transition rates \( q_{ij} = v_i P_{ij} \)

\( P_{ij}(t) \) governed by the Chapman-Kolmogorov equations and (forward) Kolmogorov differential equations

limiting probabilities found by solving steady-state equations

sufficient conditions for existence

simplified solution for birth-death processes

use limiting probabilities \( P_j \) to answer questions about system

7. Queues

\( M|M|1, M|M|k \): Poisson arrivals, exponential service times

no queue, maximum queue size, unlimited queue size

important quantities

- servers busy, servers free
- \( L, L_Q, W, W_Q \)

Little’s law: \( L = \lambda W, L_Q = \lambda W_Q \)