Review for Final

1. Probability Concepts

   conditional probabilities
   Bayes' formula
   independence

2. Random Variables

   standard distributions
   binomial, geometric, Poisson, exponential, gamma
   expected value, variance
   moment generating function
   found using conditioning

3. Markov Chains

   formulation of models, state transition diagram
   transition probabilities, higher-order transition probabilities
   absolute probabilities
   classification of states (recurrent, transient, periodic)
   limiting, stationary probabilities
   supporting theorems
   gambler's ruin
   branching processes

4. Exponential Distribution

   $f(x) = \lambda e^{-\lambda x}$, $x > 0$; $\overline{F}(x) = e^{-\lambda x}$; $E[X] = 1/\lambda$
   memoryless property, constant failure rate λ
   sum of n exponential(λ) variables is a Gamma(n, λ)
   minimum of $X_1, X_2, \ldots, X_n$ is exponential($\lambda_1 + \lambda_2 + \cdots + \lambda_n$)
   $\mathrm{P}(X_1 < X_2) = \frac{\lambda_1}{\lambda_1 + \lambda_2}$

5. Poisson Process

   $N(t)$ is the number of events occurring in $(0, t)$; λ is the average number of events per unit time
   independent and stationary increments, $N(t)$ is Poisson(λt)
   interarrival times T_i are all exponential(λ)
   waiting time $S_n = T_1 + \cdots + T_n$ is Gamma(n,λ)
   modified Poisson (filtered by probability p)
   nonhomogeneous Poisson (with parameter m defined by $\int \lambda(t)\, dt$)
   $\mathrm{P}(S_n^A < S_m^B)$

6. CTMC

   state $X(t)$, Markov property
   parameters v_i, P_{ij}, transition rates $q_{ij} = v_i P_{ij}$
   $P_{ij}(t)$ governed by the Chapman-Kolmogorov equations and (forward) Kolmogorov differential equations
   limiting probabilities found by solving steady-state equations
   sufficient conditions for existence
   simplified solution for birth-death processes
   use limiting probabilities P_j to answer questions about system

7. Queues

   $M|M|1$, $M|M|k$: Poisson arrivals, exponential service times
   no queue, maximum queue size, unlimited queue size
   important quantities
      servers busy, servers free
      L, L_Q, W, W_Q
   Little's law: $L = \lambda_a W$, $L_Q = \lambda_a W_Q$