

1. Similar to the jug pouring problem in *Die Hard*, you have two jugs initially empty. Here however they have capacities 7 and 5 gallons respectively, with no other markings on the jugs. You are able to fill or empty any jug, as well as transfer the contents between jugs (until one is empty or the other is full). We are interested in seeing which amounts can be measured out by a sequence of pourings (steps). Draw a graph representation (directed tree), starting at (0,0), showing the states that are accessible. You only need to continue as far as needed to answer the following questions.

- a. Determine a shortest sequence of steps to measure out exactly 1 gallon.
- b. Determine a shortest sequence of steps to measure out exactly 6 gallons.
- c. Is it possible to measure out all (integer amounts) of 1 through 7 gallons? Give the required (minimum) number of steps wherever possible.

2. Identify a simple function g so that $f(n) = \frac{5n^2 - n \log n}{7n - 3\sqrt{n}} = \theta(g)$ and carefully justify your answer (using simple algebra). Here and elsewhere, \log indicates logarithm to the base 2. Clearly indicate your constants C, n_0 as needed. [You can assume that $\log n \leq n$ holds for $n \geq 1$.]

3. Show that $1^4 + 2^4 + \cdots + n^4 = \theta(n^5)$. Do not try to find an explicit formula for this summation.