MthSc 814 **Problem Set #2** Fall 2009

Due 9/18/09

1. (AMO Problem 3.25) Define the graph G to have nodes (i,j) where i indicates the row and j the column in the chessboard. For example, the top row contains nodes $s = (1,1)$, $(1,2)$, $(1,3)$, $(1,6)$, and $(1,7)$.

(a) Draw the breadth-first search tree of graph G (don't draw the entire graph) that includes all nodes accessible from the source s. For consistency, always scan nodes (i,j) in increasing numerical order (that is, in order by $10i + j$).
(b) Determine a shortest sequence of moves for a knight to reach t from s.
(c) What other squares require the same minimum number of moves to be reached from node s?

2. Below are various sets S_i of consecutive integers, where each $S_i \subseteq N = \{1, 2, \ldots, 20\}$. Let the cost of set S_i be the square of its number of elements. We would like to express N as the *disjoint* union of several such sets at *minimum total cost*.

(a) Show how this can be formulated as a shortest path problem from a designated source node s to a designated sink node t. Carefully explain the construction of your network G. [HINT: You can construct G to be an acyclic network.]
(b) Use the reaching algorithm to solve this problem and thus produce a minimum cost representation of N in terms of the sets S_i. Show the sets S_i in your optimal solution as well as the minimum cost.

1	1-2	1-3	2-5	3-6	4	4-6	5-7	5-9	6-7	6-8	7-10
8-10	9-12	10-11	10-12	11-13	11-14	12-15	13-16	14-17	15	16-17	16-18
17-18	18-19	18-20	19-20	20							

3. Use Dijkstra's algorithm to find a shortest path tree rooted at node 1 in the network below. Note that two of the arcs are bi-directional. Show at each iteration $D(\cdot)$, pred$(\cdot)$, and the selected node i^*, as well as the final shortest path tree.