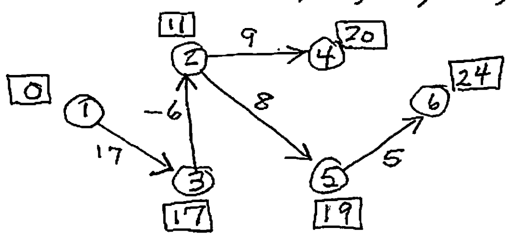
HW#3 – 814

1(a) FIFO

[top... bottom]

Step	LIST	Node Labels	Scan
1	[1]	[0, ∞, ∞, ∞, ∞, ∞]	1
2	[2, 3]	[0, 15, 17, ∞, ∞, ∞]	2
3	[3, 4, 5]	[0, 15, 17, 24, 23, ∞]	3
4	[4, 5, 2]	[0, 11, 17, 24, 20, ∞]	4
5	[5, 2, 6]	[0, 11, 17, 24, 20, 30]	5
6	[2, 6, 4]	[0, 11, 17, 23, 20, 25]	2
7	[6, 4, 5]	[0, 11, 17, 20, 19, 25]	6
8	[4, 5]	[0, 11, 17, 20, 19, 25]	4
9	[5]	[0, 11, 17, 20, 19, 25]	5
10	[6]	[0, 11, 17, 20, 19, 24]	6
11	—	[0, 11, 17, 20, 19, 24]	

SPT:



(b) PAPE

Step	LIST	Node Labels	Scan
1	[1]	[0, ∞, ∞, ∞, ∞, ∞]	1
2	[2, 3]	[0, 15, 17, ∞, ∞, ∞]	2
3	[3, 4, 5]	[0, 15, 17, 24, 23, ∞]	3
4	[2, 4, 5]	[0, 11, 17, 24, 20, ∞]	2
5	[4, 5]	[0, 11, 17, 20, 19, ∞]	4
6	[5, 6]	[0, 11, 17, 20, 19, 26]	5
7	[6]	[0, 11, 17, 20, 19, 24]	6
8	—	[0, 11, 17, 20, 19, 24]	

{ same SPT as in (a) }

2(a)

$$D^{(0)} = \begin{vmatrix} 0 & 2 & 5 & \cdot \\ 10 & 0 & \cdot & 6 \\ 7 & -4 & 0 & 3 \\ \cdot & \cdot & 3 & 0 \end{vmatrix} \qquad pred^{(0)} = \begin{vmatrix} 1 & 1 & 1 & 0 \\ 2 & 2 & 0 & 2 \\ 3 & 3 & 3 & 3 \\ 0 & 0 & 4 & 4 \end{vmatrix}$$

$$D^{(1)} = \begin{vmatrix} 0 & 2 & 5 & \cdot \\ 10 & 0 & 15 & 6 \\ 7 & -4 & 0 & 3 \\ \cdot & \cdot & 3 & 0 \end{vmatrix} \qquad pred^{(1)} = \begin{vmatrix} 1 & 1 & 1 & 0 \\ 2 & 2 & 1 & 2 \\ 3 & 3 & 3 & 3 \\ 0 & 0 & 4 & 4 \end{vmatrix}$$

$$D^{(2)} = \begin{vmatrix} 0 & 2 & 5 & 8 \\ 10 & 0 & 15 & 6 \\ 6 & -4 & 0 & 2 \\ \cdot & \cdot & 3 & 0 \end{vmatrix} \qquad pred^{(2)} = \begin{vmatrix} 1 & 1 & 1 & 2 \\ 2 & 2 & 1 & 2 \\ 2 & 3 & 3 & 2 \\ 0 & 0 & 4 & 4 \end{vmatrix}$$

$$D^{(3)} = \begin{vmatrix} 0 & 1 & 5 & 7 \\ 10 & 0 & 15 & 6 \\ 6 & -4 & 0 & 2 \\ 9 & -1 & 3 & 0 \end{vmatrix} \qquad pred^{(3)} = \begin{vmatrix} 1 & 3 & 1 & 2 \\ 2 & 2 & 1 & 2 \\ 2 & 3 & 3 & 2 \\ 2 & 3 & 4 & 4 \end{vmatrix}$$

$$D^{(4)} = \begin{vmatrix} 0 & 1 & 5 & 7 \\ 10 & 0 & 9 & 6 \\ 6 & -4 & 0 & 2 \\ 9 & -1 & 3 & 0 \end{vmatrix} \qquad pred^{(4)} = \begin{vmatrix} 1 & 3 & 1 & 2 \\ 2 & 2 & 4 & 2 \\ 2 & 3 & 3 & 2 \\ 2 & 3 & 4 & 4 \end{vmatrix}$$

2(b)

$1 \to 4$ path:

$pred(1,4) = 2$

$pred(1,2) = 3$

$pred(1,3) = 1$

$\Rightarrow$

$P: 1 \to 3 \to 2 \to 4$

$c(P) = 7$

$4 \to 1$ path:

$pred(4,1) = 2$

$pred(4,2) = 3$

$pred(4,3) = 4$

$\Rightarrow$

$P: 4 \to 3 \to 2 \to 1$

$c(P) = 9$

2(c)

$$D^{(0)} = \begin{vmatrix} 0 & 2 & 5 & \cdot \\ 10 & 0 & \cdot & 0 \\ 7 & -4 & 0 & 3 \\ \cdot & \cdot & 3 & 0 \end{vmatrix} \qquad pred^{(0)} = \begin{vmatrix} 1 & 1 & 1 & 0 \\ 2 & 2 & 0 & 2 \\ 3 & 3 & 3 & 3 \\ 0 & 0 & 4 & 4 \end{vmatrix}$$

$$D^{(1)} = \begin{vmatrix} 0 & 2 & 5 & \cdot \\ 10 & 0 & 15 & 0 \\ 7 & -4 & 0 & 3 \\ \cdot & \cdot & 3 & 0 \end{vmatrix} \qquad pred^{(1)} = \begin{vmatrix} 1 & 1 & 1 & 0 \\ 2 & 2 & 1 & 2 \\ 3 & 3 & 3 & 3 \\ 0 & 0 & 4 & 4 \end{vmatrix}$$

$$D^{(2)} = \begin{vmatrix} 0 & 2 & 5 & 2 \\ 10 & 0 & 15 & 0 \\ 6 & -4 & 0 & -4 \\ \cdot & \cdot & 3 & 0 \end{vmatrix} \qquad pred^{(2)} = \begin{vmatrix} 1 & 1 & 1 & 2 \\ 2 & 2 & 1 & 2 \\ 2 & 3 & 3 & 2 \\ 0 & 0 & 4 & 4 \end{vmatrix}$$

$$D^{(3)} = \begin{vmatrix} 0 & 1 & 5 & 1 \\ 10 & 0 & 15 & 0 \\ 6 & -4 & 0 & -4 \\ 9 & -1 & 3 & -1 \end{vmatrix} \qquad pred^{(3)} = \begin{vmatrix} 1 & 3 & 1 & 2 \\ 2 & 2 & 1 & 2 \\ 2 & 3 & 3 & 2 \\ 2 & 3 & 4 & 2 \end{vmatrix}$$

A negative cycle is detected. It can be traced out using the above pred matrix: $pred(4,4) = 2$, $pred(4,2) = 3$, $pred(4,3) = 4$ $\Rightarrow$

Cycle $4 \to 3 \to 2 \to 4$, with length -1

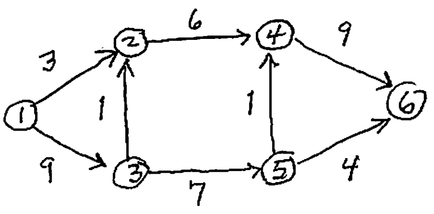
3 (a) Initial $G(\underline{x}) = G$

$v = 0$

aug. path P: 1-3-5-6

$\delta(P) = 4$

$G(\underline{x})$:

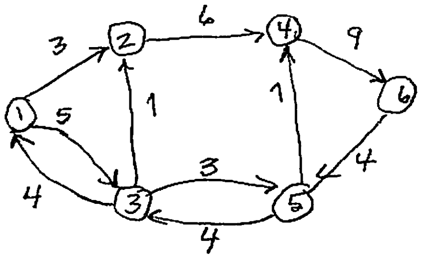


$v = 4$

aug. path P: 1-2-4-6

$\delta(P) = 3$

$G(\underline{x})$:

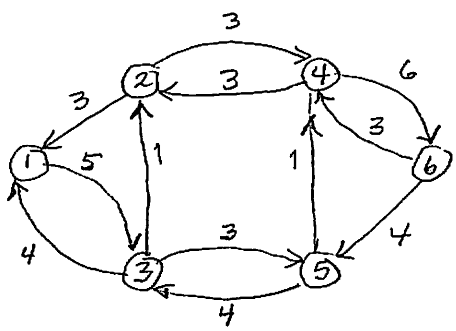


$v = 7$

aug. path P: 1-3-2-4-6

$\delta(P) = 1$

$G(\underline{x})$:

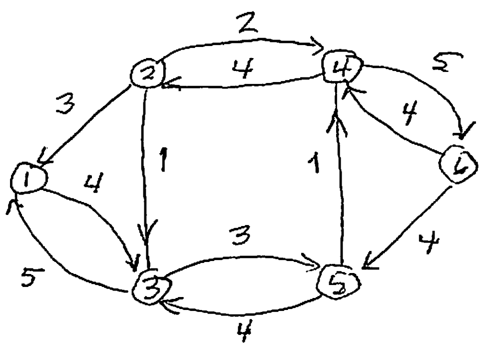


$v = 8$

aug. path P: 1-3-5-4-6

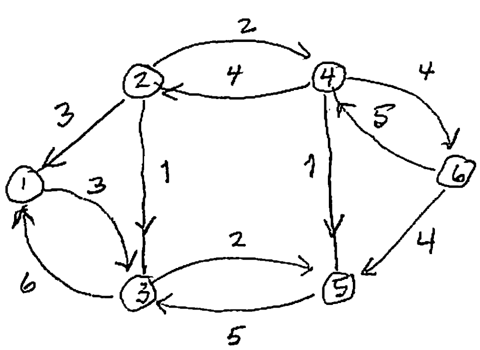
$\delta(P) = \underline{1}$

$G(\underline{x})$:

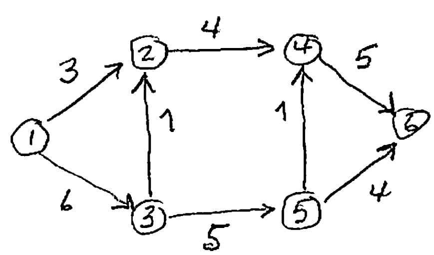


$v = 9$; no aug. path ; derive max flow

$G(\underline{x})$:

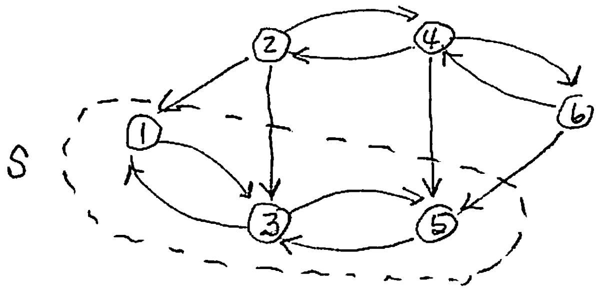


$\underline{x}^*$:



$v^* = $ max flow value $= 9$

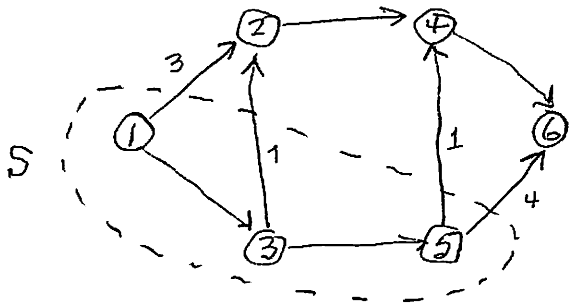
3(b) From final $G(x)$ find S = all nodes accessible from s <u>in $G(\underline{x})$</u>.



$S = \{1, 3, 5\}$

$\bar{S} = \{2, 4, 6\}$

Now look at the forward arcs of the cut $(S, \bar{S})$ <u>in G</u>:



$(S, \bar{S}) = \{(1,2), (3,2), (5,4), (5,6)\}$

$u[S, \bar{S}] = u_{12} + u_{32} + u_{54} + u_{56}$

$= 3 + 1 + 1 + 4 = 9$

Here max flow value $= v^* = 9 = u[S, \bar{S}]$ = min cut capacity, verifying the max-flow min-cut theorem.