MthSc 814 **Problem Set #4** Fall 2009

Due 11/11/09

1. An organization has five standing subcommittees $S_1 = \{A, C\}$, $S_2 = \{B, D\}$, $S_3 = \{C, E\}$, $S_4 = \{A, B\}$, $S_5 = \{D, E\}$, where A , B, …, E represent the individual members. It is desired to select a new advisory committee T of five members where each subcommittee has exactly one representative on this advisory committee. It is important to know which subcommittee each selected member is to represent.

(a) Formulate this problem as a flow problem on a network. Clearly label your network.
(b) Starting with the initial selection of A, B, C from (respective) subcommittees 1, 2, 3 plus D from subcommittee 5, apply the augmenting path algorithm to find the selection T. In the process, keep track of all augmenting paths at each step so that you can characterize **all** such valid selections T. Show your steps clearly.

2. Consider the site selection problem defined by the following table of costs and revenues:

		r_{ij}			
	c_i	1	2	3	4
1	17	–	16	4	21
2	9		–	7	13
3	28			–	16
4	19				–

(a) Draw and *fully label* a flow network that can be used to solve this problem.
(b) Start with the flow defined by $x(12,1) = 16$, $x(13,3) = 4$, $x(14,1) = 1$, $x(23,2) = 7$, $x(24,2) = 2$, $x(24,4) = 11$, $x(34,3) = 8$, $x(34,4) = 8$. Then use a shortest augmenting path algorithm [BFS] to find an optimal flow. In case of ties, choose the *lexicographically smallest* path (in terms of node labels).
(c) Find the minimum cut associated with your solution in (b) and compute its capacity. Use this minimum cut to **derive** an optimal solution to the site selection problem. Explain your reasoning.