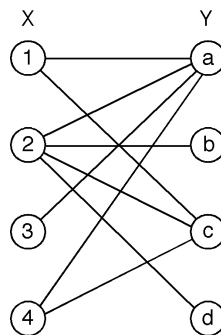


Due 11/10/11

1. In the bipartite graph shown below we are interested in finding a maximum size matching between set X and set Y .

- Set up an appropriate bipartite flow network for this problem, clearly labeled.
- Starting with the matching $(1, c)$, use the generic augmenting path algorithm to obtain a maximum size matching. At each step select a path with the *fewest* arcs; break any ties for augmenting path *lexicographically*. At each iteration, show the path selected and its capacity. Clearly display the final matching you obtain.
- From the maximum flow obtained in (b), find the associated minimum cut. List the forward arcs of this cut and compute its capacity.
- Derive from your minimum cut in (c) a node cover; explain your reasoning.
- Derive from your minimum cut in (c) a “bad” set relative to Hall’s theorem; explain your reasoning.



2. The network shown below represents a portion of an electrical power system. Nodes 1 and 2 are electricity generating plants and they produce respectively 12 and 9 megawatts. Nodes 3, 4, 5 are factories and they consume respectively 10, 5, 6 megawatts. Arc values represent capacities that limit the amount of electricity that can be transmitted along each arc.

- Set up an appropriate flow network G to model this problem. Clearly label the network.
- Find a maximum flow in G as well as a minimum cut. Use the generic augmenting path, augmenting using a path with the *fewest* arcs at each step. In the case of ties use a shortest path with *largest capacity*; if there are still ties, use the lexicographically smallest path. Verify that the max-flow min-cut theorem holds.
- Use your answers to (b) to determine if there is a way to satisfy all factory demands in the current network. If this is possible specify such an electricity transmission plan; otherwise explain why not.

