Solutions are due by 5 pm on 5/4/11. Work is to be done individually — no consultation with other persons or reference books, please.

1. **Formulate** the following as an optimization problem on a bipartite network. Show the resulting network clearly.

> At time t, a radar screen shows the (x,y) coordinates of 4 submarines; these are displayed in the first two columns of the table below, listed in order by x-coordinate. A few minutes later (at time $t + \Delta t$), the screen shows objects at the new (x,y) coordinates shown in the last two columns of the table below (also listed in order by x-coordinate). Since the 4 objects are not otherwise identified, except by their coordinates, it is desired to track these objects over time. Specifically, find an "optimal" pairing of the two sets of 4 observations, so that each pair reasonably corresponds to the same submarine, observed at the two successive times. HINT: you can use squared Euclidean distance to measure proximity.

x	y		x	y
37	19		22	40
43	70		52	37
61	64		58	16
73	25		73	43

2. **Solve** (by hand) the above problem using the appropriate algorithm developed in class.

3. Suppose that we have a connected undirected graph G and want to determine if G is in fact a bipartite graph, with the node set decomposed as $N_1 \cup N_2$. **Design** an algorithm (but do not implement as actual code) to check whether G is bipartite and if so discover the sets N_1, N_2. Pseudocode (carefully explained) will be acceptable. Also determine the worst-case time and space complexity of your proposed algorithm. Assume that G is input as an edge list.

4. An *edge cover* of a connected graph $G = (N, E)$ is a set of edges that are incident to all nodes of the graph. **Show** how you can derive a *minimum edge cover*, an edge cover with the fewest number of edges, from a related matching problem. Justify your explanation.