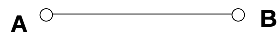
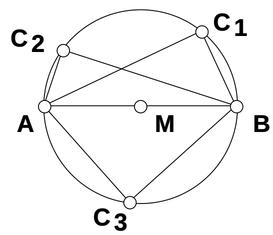
How to draw an acute scalene triangle (and others)

by Robert Simms

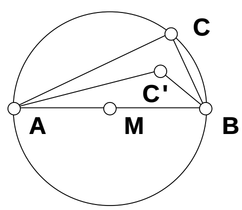
Start with a line segment. We'll call this the base of the triangle.



In order to get an acute triangle we need to avoid making a right triangle. So let's look at one way to get a right triangle. This comes from a theorem due to Euclid. The theorem states that the measure of an angle inscribed in a circle is half of the measure of the arc that is spanned (subtended) by the angle. Specifically if we place the point C on a circle with AB as diameter then the angle ACB spans a semicircle (180 degree arc) and the measure of angle ACB is 90 degrees.

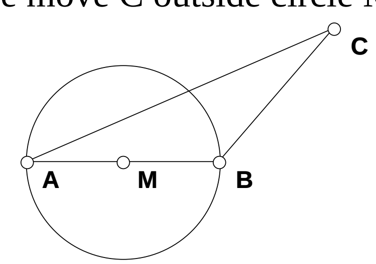


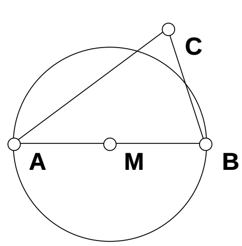
What kind of triangle do we get if we move C inside circle M?

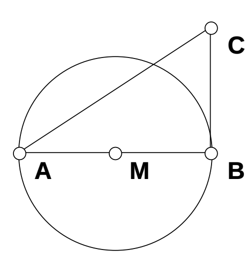


The answer is an obtuse triangle. Proof: The point C can be placed on circle M so that C' is inside triangle ABC, so we'll assume that this is the case. The measures of ABC and BAC sum to 90 degrees because m ACB is 90 degrees and all three angles of any triangle must sum to 180 degrees. Since C' is in the interior of ABC and BAC then m ABC' < m ABC and m BAC' < m BAC. So m ABC' and m BAC' sum to less than 90 degrees. Therefore, m AC'B must be greater than 90 degrees in order to have the angle sum in triangle ABC' be 180 degrees. Hence, triangle ABC' is obtuse.

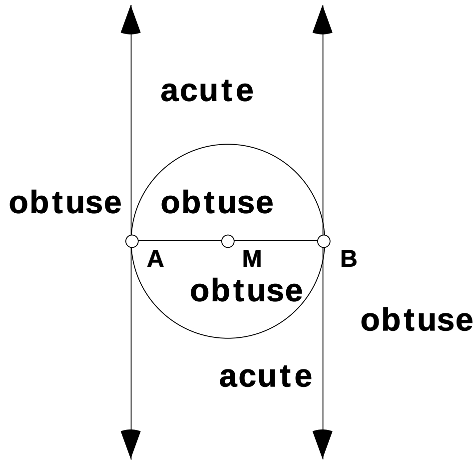
What if we move C outside circle M? There are a few possibilities.





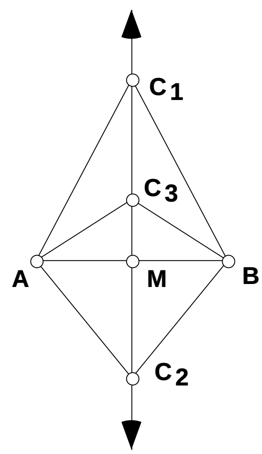


If we choose C on a line perpendicular to side AB through A or through B then the triangle ABC will be right. Observe, in the figure below, how the perpendicular to side AB through A and the perpendicular to side AB through B serve as boundaries between acute and obtuse locations for C. This completes the boundaries between regions corresponding to acute and obtuse triangles that have segment AB as a side.

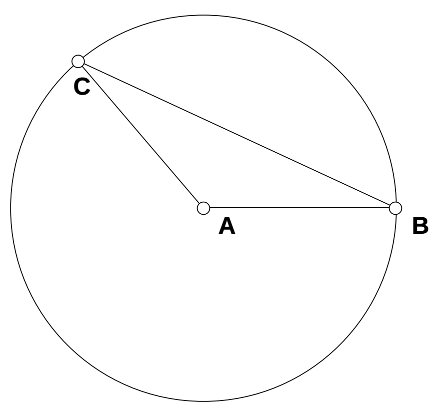


But how can we get an acute scalene triangle? The key is to choose the third vertex in an acute region and avoid making the triangle isosceles.

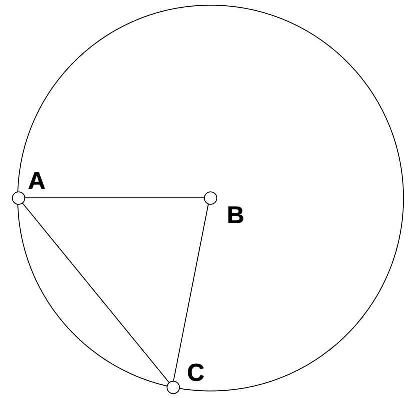
One way to get an isosceles triangle is to place C on the perpendicular bisector of AB. This makes length AC the same as length BC, which can be proved using SAS and corresponding parts.



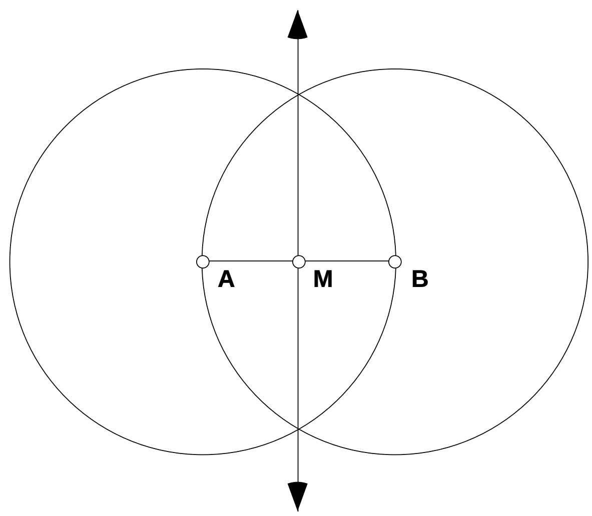
We would also have an isosceles triangle if side AC has the same length as side AB. Use side AB as a radius to sweep out a circle about A and use a point on the circle for C. Then we'll have AC = AB.



We can do the same thing about point B to get BC = AB.

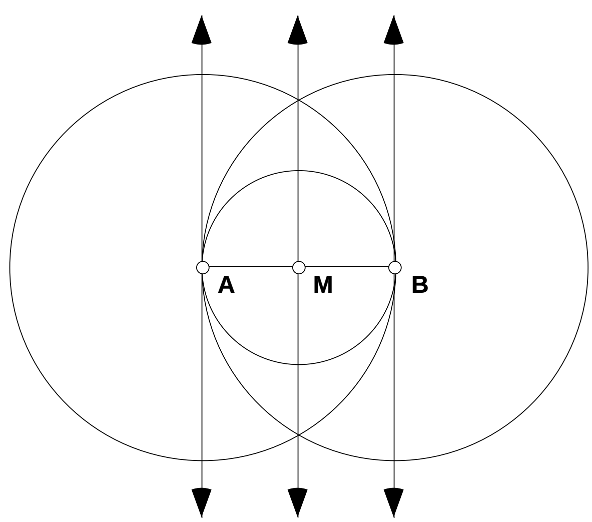


So it's on the following three curves (two circles and one line) that we can place C to make triangle ABC isosceles.



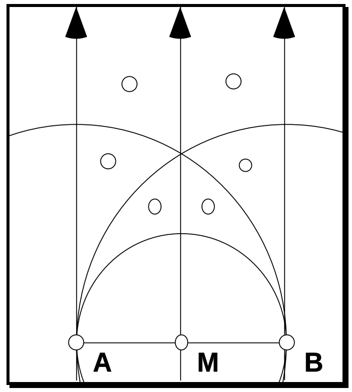
Additionally, if we place C at an intersection of the two circles we get an equilateral triangle. That's because we'll have both AC=AB and BC=AB for the same point C.

Putting the right and isosceles locations for C together, we arrive at this diagram:

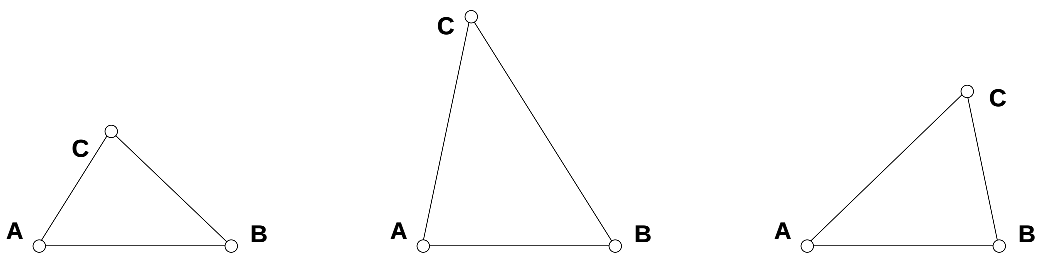


The curves in the above diagram divide the acute and obtuse regions into subregions.

So now we can answer the question of how to draw an acute scalene triangle. Place C in the interior of any acute subregion. To make a triangle that looks really acute and scalene, choose C so that it is not too close to a right or isosceles boundary of the subregion. The picture below shows just such choices, above segment AB.



Here are three acute scalene triangles made by choosing a third vertex from the interiors of three different acute subregions.



The points were chosen so that the three triangles are similar to each other. To see this, rotate, in your mind, one of these triangles so that a different side is the base. You'll notice that the triangle will strongly resemble one of the other triangles because they have matching angles in the same positions relative to their bases.

Given any triangle with a side matching segment AB, there are two other ways to find similar triangles with the same property. One way is to reflect the triangle in the perpendicular bisector of segment AB. The other way is to reflect the triangle in the line AB (the extension of segment AB). Technically speaking, the triangles made in these ways are congruent to the original triangle, but congruent triangles are similar.

These two reflections together with the similarity property described above, reduce the problem of choosing a third vertex for an acute scalene triangle down to choosing a third vertex from just one acute subregion. In other words, given any acute scalene triangle, you can always find a point in any acute subregion that gives you a similar triangle.

This similarity argument can be applied to right and obtuse triangles as well. But don't stop there -- see what else there is to discover!

For more information, visit http://www.math.clemson.edu/~rsimms/triangle/

Last modified 16 Oct 1997