Statistical functions for CASIO Calculator

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1 Introduction.

In this document we suggest some improvements and additions to the statistical functions already in place in the CASIO CFX-9850Ga plus calculator. The main additions considered are two-way ANOVA and multiple linear regression. We also suggest in general that residuals (from regression as well as ANOVA) are automatically stored in a list when a model is fit and that Mean Squared Error (MSE) is reported for each fitted model.

We begin with some general comments about statistical model fitting and the regression functions already in place in the calculator . We will then discuss the implementation of a general ANOVA framework which can be used for both one-way and two-way ANOVA, and can be easily generalized to include multi-factor ANOVA models. For multiple regression we suggest that the MSE and the adjusted coefficient of multiple determination are reported for each fitted model. The standard graphical diagnostics for linear models can be performed using graphical procedures already in place in the calculator.

2 Model Assessment.

Two important aspects of statistical model fitting which are sometimes overlooked in introductory courses are model choice and model assessment. From a practical point of view both rely heavily on data display.

An important feature of the CASIO CFX-9850Ga is the ability to display a fitted regression curve over a scatter diagram. This can be used to visually assess the fit of the chosen regression function. The calculator can quickly fit competing regression functions and the graphs can be used to subjectively to compare fits.

A more objective measure of model fit should be reported for each fit. The usual measure of fit is mean squared error (normalized sum of squared residuals), which measures the distance of the fitted values from the observed. The least squares fit is an attempt to minimize the expected mean squared error (MSE). The function which provides the smallest MSE is usually selected to model the data. The calculator can already be used to calculate residuals and store them in a list. This list can be used to calculate MSE. Once the residuals are stored in a list the graphical displays already included in the calculator can be used to check for normality and look for functional relationships between the dependent variable and the residuals.

In general statistical model fitting includes three steps: examining collected data, fitting a model, and diagnostic procedures to examine the model assumptions. Many of the classic statistical tests including ANOVA and tests for regression slope depend on the assumption of normality. Given any fitted model this assumption can be checked through a normal probability plot of the residuals. The CASIO CFX-9850Ga+ will calculate residuals from a fitted regression, but it would be nice if the residuals were calculated and stored in a list when any model was fit.

3 ANOVA

For a two factor experiment we consider an analysis at the level of Moore and McCabe (1999). The steps of the process considered are data input, graphical display and display of test results.

Data input. The first thing to be considered is user input. One possibility is to have a list for each factor (to indicate the level) and then a list for the dependent variable. The list corresponding to each factor would consist of a categorical variable indicating the level of the factor.

As an example we consider a two factor experiment measuring reaction time of air traffic control personnel to 5 different emergency conditions (factor A) displayed on 3 different display panels (factor B). The data is given in Coffin and Nelson (1999). For the air traffic control data, the statistical data list would be as follows.

	list 1	list 2	list 3
1	1	1	18
2	3	5	28
⋮	⋮	⋮	⋮

The second row indicates the controller response was 28 seconds when display panel 3 was used under emergency condition 5. Once the data is entered the user can perform the ANOVA test.

Keeping the set up similar to the CFX-9850Ga+ the user could go into test, then ANOVA. The current display for a one-way ANOVA test with 2 levels is

```
ANOVA
How many:2
list 1 : list1
list 2 : list2
```

where "How many" refers to the number of *levels* of the one factor. This could be replaced with

```
ANOVA
How many: 1
Factor A:  list1
Dependent: list2
```

with list1 consisting of a categorical variable indicating the level of the factor and list2 consisting of the corresponding measurements on the dependent variable. The display for the two-way ANOVA would then be

```
ANOVA
How many: 2
Factor A:  list1
Factor B:  list2
Dependent: list3
Interaction: Y/N
Execute:
```

with list1 indicating the level of factor A and list2 indicating level of factor B. In this set up "How many" is referring to the *number of factors*, while under the old set up it was asking about the number of levels of the single factor. The levels are coming from the list of categorical variables (list1 and list2). The user must indicate whether to test for interaction or not. This general input can easily be generalized to multi-factor ANOVA if this becomes desirable in a future version of the calculator.

Graphical display. The analysis of data from a designed experiment usually begins with graphical display. Two useful displays are error-bar charts for one-factor experiments and interaction plots (discussed below) for two-factor experiments (see Coffin and Nelson, 1999). The ANOVA test is used to test for difference among the treatment groups. If a significant difference is detected, contrasts are used to isolate the source of the difference. In the case of a two factor experiment an interaction plot can be used to interpret the results of the ANOVA test.

An important issue for a two factor experiment is the presence of interaction. If there is a combined affect of the two factors the interpretation of the results of a two-way ANOVA changes. An interaction plot is the first step. To make an interaction plot the means for each combination of factor level must be calculated. For the display panel data the interaction plot is given in figure 1. If the formal test (ANOVA) indicates significant interaction between the two factors, an interaction plot can be used to interpret the results (see e.g. Moore and McCabe, 1999, Example 13.7). The interaction plot should be an option in the ANOVA test menu.

Displaying results. When the ANOVA test is executed the results should be displayed in a standard ANOVA table. The ANOVA table for the air traffic control data for the model including interaction is shown below.

source	df	ss	ms	F	P-value
Panel (A)	2	1227.8	613.9	86.9	0.0000
condi (B)	4	2850.1	712.5	100.8	0.0000
A*B	8	44.9	5.6	0.8	0.6167
resid	15	106.0	7.1		
Total	29	4228.8			

4 Multiple Regression.

There are many suggested strategies for fitting multiple regression models. Most consist of fitting various candidate models and picking the model which fits best according to some criteria (e.g. see Neter, Wasserman, and Kutner, 1990; Chapter 12). For each fitted model it is necessary to report some standard measures of model fit. Standard residual diagnostics (usually graphical) can be used to test model assumptions such as normality and constant variance of error terms.

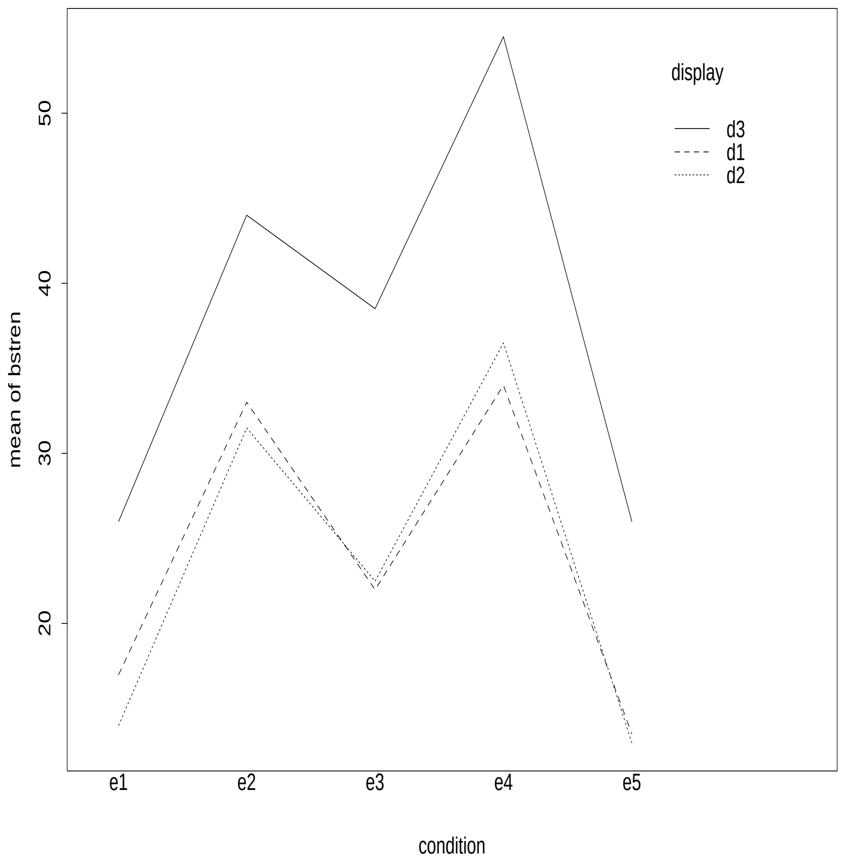

Figure 1: Interaction plot for 2-factor experiment.

The two most common numerical measures used to compare regression fits are the coefficient of multiple determination and MSE (see Neter, Wasserman, and Kutner, 1990; Chapter 7). For a model with p independent variables

$$MSE = \frac{e^t e}{(n-p)},$$

where n is the sample size, e is the vector of residuals, and t denotes transpose. The coefficient of multiple determination, denoted by R^2, measures the proportionate reduction in total variation in the dependent variable Y associated with a given set of predictor variables. This coefficient can be used to compare fitted regression models. It can calculated as the squared correlation coefficient between the dependent variable Y and the fitted values $\hat{Y}$. A problem with R^2 is that it will never be made smaller by adding a variable to the model. We suggest including the *adjusted coefficient of multiple determination*

$$R_a^2 = 1 - \frac{MSE}{S^2},$$

where S^2 is the sample variance of the dependent variable.

The residuals from a fitted model can be used to check model assumptions and look for relationships (between predictor variables and the dependent variable) not captured in the model. Once the residuals have been stored in a list, the graphical procedures already in place in the CFX-9850Ga+ can be used to make appropriate plots. The two types of residual plots which are of interest are scatterplots and normal probability plots. A scatterplot of the residuals against an independent variable (say X_i) should not be systematic if the error variance is constant and if the information in X_i about Y is incorporated in the model. A normal probability plot of the residuals will reveal significant departures from the assumption of normality.

Fitting a regression model. In fitting a multiple regression model two things need to be decided: what independent variables should be used and what is the appropriate regression function for that set of variables. Whatever procedure is being used to pick the "best" set of regressors, Forward Stepwise, Backward stepwise, All-Possible-Regressions, etc..., the procedure consists of fitting many models and comparing fits. It is therefore enough to build into the calculator the ability to fit and assess the fit of one single regression model. The steps for fitting a model using a given set of regressors are

1. Fit a model and store residuals in a list.
2. Plot the residuals against each independent variable to look for departures from constant variance and/or systematic change of residuals with changes in the independent variable
3. Make a normal probability plot from the residuals

If in the second step the error variance appears non constant, the dependent variable must be transformed and the model refit on the transformed data. If a plot of an independent variable against the residuals is systematic than the independent variable should be transformed and the model refit. If the normal probability plot reveals a significant departure from normality, the dependent variable should be transformed.

5 CONCLUSIONS.

We suggest the following

1. Store the residuals in a list for both regression and ANOVA
2. Change the user input for ANOVA (categorical variables).
3. Include interaction plot under ANOVA menu.
4. Report ANOVA results in a standard ANOVA table.
5. Calculate and report MSE for every fitted model.
6. Report R_a^2 for a fitted regression model.

References

[1] Coffin and Nelson (1999), *Introductory Statistics for Scientests and engineers.* (technical report, Dept. of Mathematical Sciences, Clemson University)

[2] Moore and McCabe (1999), *Introduction to the Practice of Statistics* (W.H. Freeman and Company, third edition).

[3] Neter, Wasserman and Kutner (1990), *Applied Linear Statistical Models.* (Irwin, Boston, third edition).